

# Two-port network model of fixed-speed wind turbine generator for distribution system load flow analysis

Rudy Gianto\*, Kho Hie Khwee, Hendro Priyatman, Managam Rajagukguk

Department of Electrical Engineering, Tanjungpura University,  
Prof. Dr. H. Hadari Nawawi St., Pontianak 78124, Indonesia

\*Corresponding author, e-mail: rudygianto@gmail.com

## Abstract

Load flow analysis has always been used in determining the steady-state operation of an electric power or distribution system. For conventional power system without wind turbine generator, the method for load flow analysis has been well established. However, for modern system embedded with wind turbine generator, the investigation of analysis method is still an active research area. This paper proposed a new method to integrate fixed-speed wind turbine generator into distribution system load flow analysis. The proposed method is derived based on two-port network theory where the parameters of induction generator of the wind turbine generator are embedded in general constants of the two-port network. The proposed method has been tested and verified using a representative electric distribution system.

**Keywords:** distribution system, load flow, two-port network, wind turbine

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## 1. Introduction

It has widely been known that the steady-state operation of a power grid is usually evaluated by load flow analysis. For conventional power system without wind turbine generating system (WTGS), the method for load flow analysis has been well established. However, for modern power system containing WTGS, the method of analysis is still under development. To perform the load flow analysis for system with WTGS, the first step is to develop a valid model for the WTGS. After the model has been developed, the next step is to incorporate the model into the analysis. The load flow problem is then solved, and the steady-state operation of the system (including the WTGS) can be evaluated properly. Some researchers have investigated the WTGS modeling and its integration into load flow analysis [1-18]. In [1-15], a mathematical model of a WTGS has been proposed. The equations from the model are then combined with the equations arising from load flow formulation of the system without WTGS. The whole set of equations is then iteratively solved to obtain the load flow solution. However, the proposed WTGS model is quite complicated, and therefore, its application in power or distribution system containing multiple WTGS can be difficult.

In [16-18], a method has been proposed to incorporate fixed-speed WTGS into a standard load flow program. The proposed model in [16-18] adds two buses, two series elements, one shunt element and one load to the existing power network. Obviously, the addition will increase the size of the power system and also the number of equations in the load flow formulation. As a consequence, the load flow problem will be more difficult to solve, especially if the system has many WTGSs. In the present paper, the proposed model for fixed-speed WTGS is based on two-port network model. In the model, the parameters of induction generator of the WTGS are embedded in general constants of the two-port network. The WTGS mathematical model is then derived based on the equations of two-port network theory. The proposed model is simpler and can easily be integrated into the power or distribution system load flow analysis. Moreover, its application in the system containing multiple WTGSs is also simple and straightforward. Results of the proposed model validation and verification using a representative test system, i.e. 33-bus distribution network, are also presented in this paper.

## 2. Research Method

### 2.1. Formulation of Distribution System Load Flow Problem

In distribution system load flow (or power flow) analysis, the study is carried out to determine the following quantities: (i) bus or nodal voltages, and (ii) substation power. These quantities can be calculated by solving a set of nonlinear equations as follows:

$$\mathbf{S}_G - \mathbf{S}_L - \text{diag}(\mathbf{V})\mathbf{Y}^* \mathbf{V}^* = \mathbf{0} \quad (1)$$

In (1),  $\mathbf{S}_G$ ,  $\mathbf{S}_L$ ,  $\mathbf{V}$  and  $\mathbf{Y}$  are generated power vector, load power vector, nodal voltage vector and nodal admittance matrix, respectively. For n-node power system, these vectors and matrix are of the forms:

$$\mathbf{S}_G = [S_{G1} \ S_{G2} \ \Lambda \ S_{Gn}]^T \quad (2a)$$

$$\mathbf{S}_L = [S_{L1} \ S_{L2} \ \Lambda \ S_{Ln}]^T \quad (2b)$$

$$\mathbf{Y} = \begin{bmatrix} Y_{11} & Y_{12} & \Lambda & Y_{1n} \\ Y_{21} & Y_{22} & \Lambda & Y_{2n} \\ M & M & O & M \\ Y_{n1} & Y_{n2} & \Lambda & Y_{nn} \end{bmatrix} \quad (2c)$$

where:

$S_{Gi}$  : generated power flowing into node i

$S_{Li}$  : load power flowing out node i

$V_i$  : voltage at node i

$Y_{ij}$  : element ij of admittance matrix

After (1) has been solved and all of the quantities have been determined, the line flows and losses can also be calculated. It is to be noted that distribution system is usually fed at one bus (substation bus), and the voltage at this bus is known or specified. Therefore, only the voltages at the remaining buses (load buses) need to be computed. Table 1 shows the known and unknown quantities of the distribution system load flow formulation. For conventional system without WTGS, (1) is the only set of equations that needs to be solved to obtain the load flow solution. However, for modern system with WTGS, additional equation is needed to facilitate and incorporate the WTGS into the load flow analysis. The modeling and integration of the WTGS will be discussed in the following section.

## 2.2. Proposed Model of WTGS and Its Integration

### 2.2.1. Two-Port Network Theory

In the present work, development of fixed-speed WTGS model will be based on two-port network mathematical model where nodal power equations are used in the model derivation. The general two-port network containing passive impedances is presented in Figure 1. In two-port network theory, it can be shown that the voltage/current relationship is of the form:

$$\begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} = \begin{bmatrix} E & F \\ G & H \end{bmatrix} \begin{bmatrix} V_1 \\ -I_1 \end{bmatrix} \quad (3)$$

where:

$$\begin{bmatrix} E & F \\ G & H \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1} \quad (4)$$

In (4), A, B, C and D are the general two-port network constants. The values of these constants depend on the network components, i.e. their impedances and admittances. The (3) is equivalent to the following two (5a) and (5b). The development of the proposed WTGS model based on (5) will be explained in the next section.

$$V_2 = EV_1 - FI_1 \tag{5a}$$

$$I_2 = -GV_1 + HI_1 \tag{5b}$$

Bus Type	Known Quantity	Unnown Quantity
Substation	$S_{Li}$ and $V_i = 1.0$	$S_{Gi}$
Load	$S_{Li}$ and $S_{Gi} = 0$	$V_i$

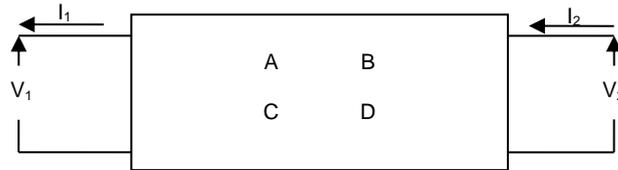


Figure 1. General two-port network

**2.2.2. Proposed Model of WTGS**

Figure 2(a) shows fixed-speed WTGS connected to a distribution system [16-20]. Power converter of the WTGS is squirrel cage induction generator (SCIG). The SCIG has mechanical power input  $P_m$  and electrical power output  $S_g = P_g + jQ_g$  as can be seen in Figure 2(b).

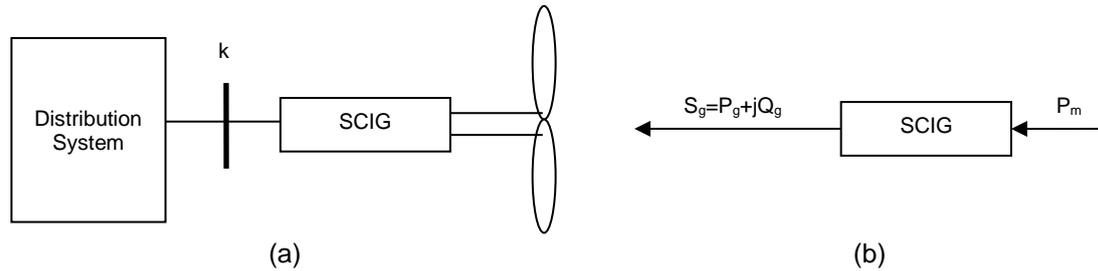


Figure 2. WTGS connected to distribution system

Figure 3(a) shows steady-state equivalent circuit of the SCIG [21-23] where  $R_1$ ,  $X_1$ ,  $R_2$ ,  $X_2$ ,  $R_c$  and  $X_m$  denote stator resistance, stator leakage reactance, rotor resistance, rotor leakage reactance, core loss resistance and magnetic reactance, respectively.  $R_2(1-s)/s$  is the dynamic resistance and its value depends on slip  $s$ . Power of the dynamic resistance represents the mechanical power  $P_m$  delivered by the wind turbine to SCIG. The value of this power depends mostly on the wind speed and it can be determined using the power curve provided by the turbine manufacturer. In the proposed method, the WTGS model is obtained by viewing the SCIG equivalent circuit of Figure 3(a) as the two-port network depicted in Figure 3 (b). In Figure 3 (b), the impedances  $Z_1$ ,  $Z_2$  and  $Z_3$  are given by:

$$\begin{aligned} Z_1 &= R_1 + jX_1 \\ Z_2 &= R_2 + jX_2 \\ Z_3 &= jR_c X_m / (R_c + jX_m) \end{aligned} \tag{6}$$

it is to be noted that for the two-port network of Figure 3 (b), the formulas for general network constants are of the forms [24]:

$$\begin{aligned}
 A &= 1 + Z_1 / Z_3 \\
 B &= Z_1 + Z_2 + Z_1 Z_2 / Z_3 \\
 C &= 1 / Z_3 \\
 D &= 1 + Z_2 / Z_3
 \end{aligned}
 \tag{7}$$

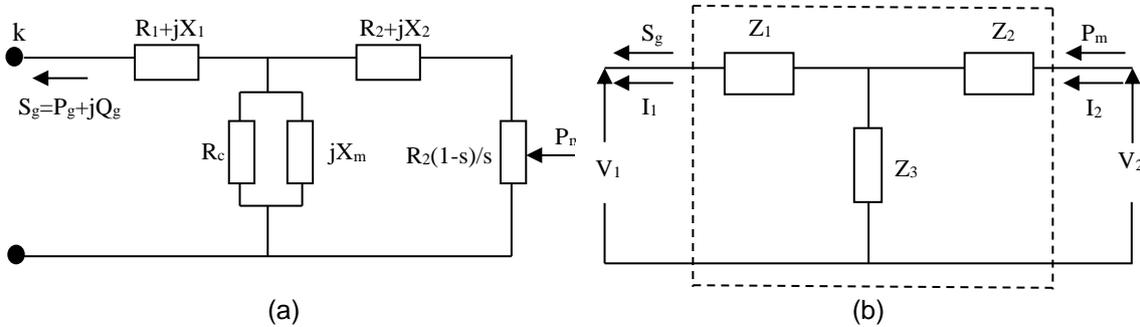


Figure 3. Steady-state equivalent circuit of SCIG

based on Figure 3 (b), the SCIG electrical power output is formulated as:

$$S_g = P_g + jQ_g = V_1 I_1^* \tag{8}$$

and the mechanical power delivered by wind turbine to the SCIG is calculated as:

$$S_m = P_m + j0 = V_2 I_2^* \tag{9}$$

Application of two-port network equations in the above SCIG mechanical power input equation, or by substituting (5a) and (5b) into (9), gives:

$$S_m = (E V_1 - F I_1)(-G^* V_1^* + H^* I_1^*) \tag{10}$$

or:

$$S_m = -E G^* V_1 V_1^* + E H^* V_1 I_1^* + F G^* V_1^* I_1 - F H^* I_1 I_1^* \tag{11}$$

on using (8) in (11), i.e. substituting  $I_1$  in (11) by  $S_g^*/V_1^*$ , the following equation is obtained:

$$S_m + E G^* V_1 V_1^* - E H^* S_g - F G^* S_g^* + F H^* \frac{S_g S_g^*}{V_1 V_1^*} = 0 \tag{12}$$

In (12) is the proposed model for fixed-speed WTGS. It is to be noted that  $V_1$  in (12) is the voltage at WTGS bus (or it is equal to  $V_k$  in Figure 3 (a)). Incorporation of the model into the load flow formulation (1) will be discussed in the next section.

**2.2.3. Integration of WTGS Model**

For modern system containing fixed-speed WTGS, solution to the power flow problem can be found by simultaneously solving (1) and (12). It can be seen that (12) is the additional equation to the formulation in (1). Whereas, the additional quantity that need to be calculated is the WTGS power output  $S_g$ . Table 2 shows the known and unknown quantities in the load flow formulation of the system with WTGS. It is to be noted that since sets of the equations, i.e

(1) and (12), are nonlinear; they are usually solved using iterative technique (for example: Newton-Raphson method).

### 3. Results and Analysis

#### 3.1. Test System

The proposed method for incorporating WTGS in distribution system load flow analysis is tested by using the 33-bus system [25]. This distribution network has the system voltage of 12.66 kV. One line diagram of the distribution system is shown in Figure 4, and the detail of the system data can be found in [25]. In the present work, it is assumed that the distribution system has one WTGS and it is connected to bus 33. Data for the SCIG of WTGS is shown in Table 3. All of the data are in pu on 1 MVA base. Results of the load flow analysis for the test system are presented in the following section.

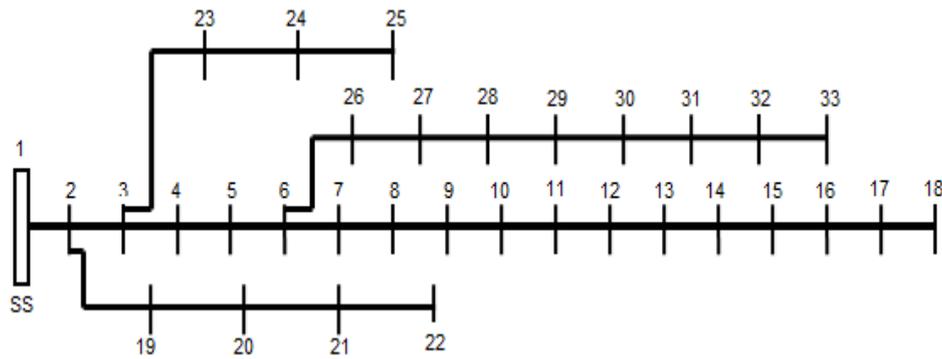


Figure 4. 33-Bus distribution network

Table 2. Bus Types and Quantities for System with WTGS

Bus Type	Known Quantity	Unnown Quantity
Substation	$S_{Li}$ and $V_i = 1.0$	$S_{Gi}$
Load	$S_{Li}$ and $S_{Gi} = 0$	$V_i$
WTGS	$S_{Li}$ and $S_{Gi} = 0$	$S_{Gi}$ and $V_i$

Table 3. SCIG Data

Parameter	R1	X1	R2	X2	Rc	Xm
Value	0,01	0,05	0,01	0,05	100	5

#### 3.2. Results and Analysis

In this paper, power flow calculations were carried out for various values of mechanical power  $P_m$ . The mechanical powers ranging from 0.1 to 1.0 pu were taken in the investigation. These values represent the low speed and higher speed wind conditions. It is to be noted that all of the computations were done on PC, and the proposed method were implemented as MATLAB codes (m-files).

Results of the calculations (in terms of WTGS voltage/power and substation power) are shown in Table 4. It is to be noted that the results of the proposed method are accurate and in excellent agreement with those of the method in [16-18]. These results confirm that the proposed two-port model is valid and can be used as a method for incorporating fixed-speed wind turbine generator into distribution system load flow analysis.

Since fixed-speed WTGS is usually equipped with shunt capacitance to support the reactive-power consumed by induction generator of the WTGS, the effects of the capacitance installation are also investigated in this paper. Table 5 shows the results of load flow analysis (in terms of WTGS voltage/power and substation power) when the capacitance with the capacity of 0.5 pu is installed. To observe the effects more clearly the results are also presented in graphical forms in Figures 5 and 6.

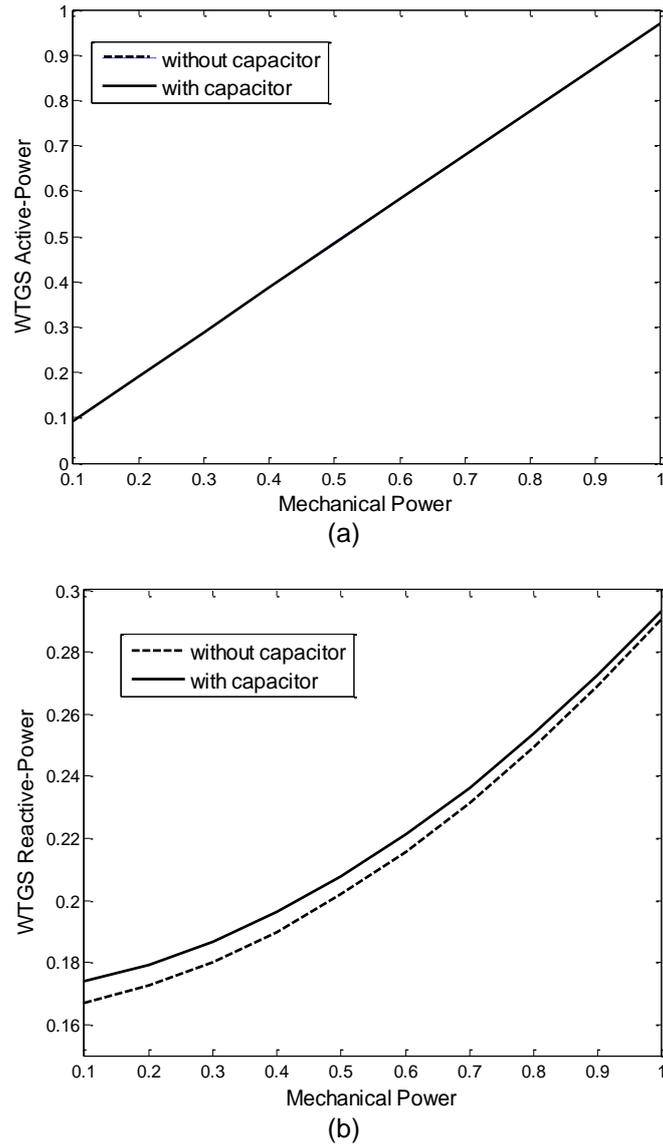


Figure 5. WTGS power versus mechanical power

Table 4. WTGS Voltage/Power and Substation Power

Pm	WTGS Voltage	WTGS Power	Substation Power
0,1	0.91419	0.0912-j0.1670	3.8423+j2.6135
0,2	0.91866	0.1904-j0.1724	3.7326+j2.6140
0,3	0.92296	0.2892-j0.1801	3.6249+j2.6158
0,4	0.92710	0.3875-j0.1899	3.5190+j2.6208
0,5	0.93108	0.4854-j0.2018	3.4148+j2.6289
0,6	0.93492	0.5828-j0.2157	3.3124+j2.6400
0,7	0.93861	0.6799-j0.2315	3.2116+j2.6540
0,8	0.94217	0.7765-j0.2493	3.1125+j2.6709
0,9	0.94559	0.8728-j0.2689	3.0149+j2.6906
1,0	0.94888	0.9687-j0.2904	2.9189+j2.7131

Table 5. WTGS Voltage/Power and Substation Power for System with Capacitance

Pm	WTGS Voltage	WTGS Power	Substation Power
0,1	0.93305	0.0909-j0.1738	3.7969+j2.0911
0,2	0.93744	0.1901-j0.1792	3.6874+j2.0895
0,3	0.94167	0.2889-j0.1866	3.5796+j2.0911
0,4	0.94575	0.3873-j0.1961	3.4735+j2.0956
0,5	0.94968	0.4852-j0.2076	3.3689+j2.1031
0,6	0.95348	0.5828-j0.2211	3.2660+j2.1133
0,7	0.95715	0.6799-j0.2364	3.1645+j2.1264
0,8	0.96069	0.7767-j0.2536	3.0645+j2.1421
0,9	0.96411	0.8731-j0.2726	2.9659+j2.1604
1,0	0.96740	0.9692-j0.2934	2.8688+j2.1814

Table 5 clearly shows that shunt capacitance is able to support the WTGS reactive-power demand as indicated by the improvement of WTGS voltage profile. Although its value is slightly less than mechanical power input (due to losses in induction generator),

active-power generation of WTGS is always proportional to the mechanical power input as shown in Figure 5 (a). However this is not the case for the variation of WTGS reactive-power. With the increase in mechanical power, the WTGS reactive-power increases almost exponentially as shown in Figure 5 (b). In other words, the increase in WTGS active-power generation requires higher increase in WTGS reactive-power demand. This result is also confirmed by Figure 6 (b). Figure 6 (a) shows that with the increase in mechanical power, the active-power supplied by distribution substation decreases linearly. This result is expected because with the increase in mechanical power, the WTGS active-power generation also increases, and therefore more active-power can be delivered by WTGS to support the system load demand. Consequently, with the increase in WTGS mechanical power, distribution substation can deliver less active-power since part of the system load is supplied by WTGS.

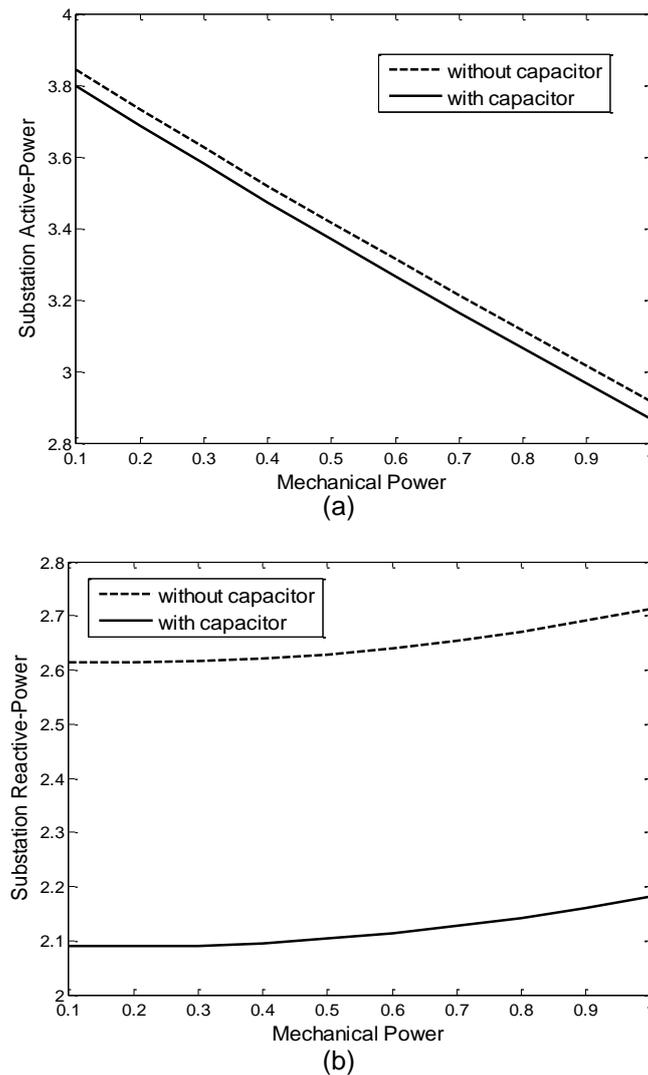


Figure 6. Substation power versus mechanical power

#### 4. Conclusion

A simple method for incorporating fixed-speed WTGS into a distribution system load flow analysis has been presented in this paper. The development of the proposed method is based on two-port network theory where equations from the electric circuit theory have been utilized to derive the proposed WTGS mathematical model. By integrating the model into the load flow analysis, the steady-state operations of the system (including the WTGS) can then be

evaluated. The proposed method has been tested and verified using a representative test system, i.e. 33-bus distribution network.

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