First Order Space Time Autoregressive Stationary Model on Petroleum Data

K Joebaedi\textsuperscript{1*}, K Parmikanti\textsuperscript{2}, Badrulfalah\textsuperscript{3}

\textsuperscript{1}Mathematics Department, FMIPA Universitas Padjadjaran, Bandung, Indonesia
\textsuperscript{2}Mathematics Department, FMIPA Universitas Padjadjaran, Bandung, Indonesia
\textsuperscript{3}Mathematics Department, FMIPA Universitas Padjadjaran, Bandung, Indonesia

*E-mail : khafsah.joebaedi@unpad.ac.id

Abstract. First order Space-Time Autoregressive model is one of the models which involves location and time. STAR(1;1) model stationary can be used to forecast future observation at a location based on one previous time of its own location and the spatial neighborhood. STAR(1;1) model on petroleum productivity data in Balongan, Indramayu, West Java with eigenvalue less than 1. It indicates that STAR (1;1) model on petroleum productivity data in Balongan, Indramayu, West Java meets the stationary requirement.

Keywords: autoregressive, petroleum data, STAR, first order

1. Introduction

Space-Time Autoregressive (STAR) model is the development of a univariate time series model into a multivariate time series model\textsuperscript{1-4}. The STAR(1;1) model states that the current time observation at a particular location is influenced by the observation of previous time at the location and its spatial neighborhood which are a part of the same group \textsuperscript{13}. For the simplicity of the model, the study will be focused on time lag 1 and spatial lag 1. A simple, stationary time series model is first order Autoregressive model \textsuperscript{5-11} : AR(1). AR(1) model developed into Vector Autoregressive(1) model: VAR(1), first order Space Time Autoregressive(1) model: STAR(1;1)\textsuperscript{2}. Petroleum production in Balongan, Indramayu, West Java is a time series problem, which can be modeled by AR(1) model, VAR(1) model, and STAR(1;1) model. \textsuperscript{12} Providing procedures in the form of; identification, parameter estimation, and diagnostic examination.

2. Methods used

Applying STAR (1;1) stationary model on petroleum data in Balongan, Indramayu, West Java. The STAR(1;1) model states that the current time observation at a particular location is influenced by the observation of previous time at the location and its spatial neighborhood which are a part of the same research group, thus STAR(1;1) model is applied on petroleum data, specifically on well-1 and well-3 petroleum drilling data in Balongan, Indramayu, West Java. For the simplicity of the model, the study will be focused on time lag 1 and spatial lag 1 on STAR(1;1) model at several lag locations. And the STAR(1;1) model is examined or assessed. After the assessment phase, model validation is conducted through error. If the model determined as adequate, then it can be used to forecast future observation on petroleum data in Balongan, Indramayu, Jawa Barat.
3. Results and Discussion

Bivariate VAR(1) model is said to be first order stationary, if the mean and covariance are not dependent on \( t \):

\[
\begin{align*}
E[z(t)] &= \mu \\
\text{Cov}[z(t), z(s)] &= E[z(t)z(s)'] = E[z(0)z(s-t)'] \\
&= C(0, s-t) = C(t-s) = C(s-t)
\end{align*}
\]  

From equation (1) covariance matrix \( C \) only depends on time shift or time lag \( (t-s) \) [5] in [3]. The theorem of VAR(\( p \)) model stationary requirement with \( u = 1, 2, \ldots N \) variate, presented [5] in [3] as follows:

If \( x_u \) equation solution

\[
|x_u| \sum \Phi (j) x_u = 0
\]

located in a unit circle (\( |x_u| < 1 \)), then the VARMA(p,q) process is stationary.

Next [6] provides Hannan stationary requirement equivalence above for VAR(1) model, as follows: If \( \forall B \in C, |B| > 1 \) applies \( |I - \Phi B| = 0 \), then VAR(1) is stationary. In other words, the B roots of \( |I - \Phi B| = 0 \) are outside the unit circle.

STAR(1;1) model by[4] in[3] is stated:

\[
z(t) = z(t-1) + Wz(t) + e(t)
\]  

\[
W = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}
\]
equal weight

STAR(1;1) model equation for 2 locations can be presented as follows:

\[
\begin{bmatrix} z_1(t) \\ z_2(t) \end{bmatrix} = \phi_{01} \begin{bmatrix} z_1(t-1) \\ z_2(t-1) \end{bmatrix} + \phi_{11} \begin{bmatrix} w_{11} \\ w_{21} \end{bmatrix} \begin{bmatrix} z_1(t-1) \\ z_2(t-1) \end{bmatrix} + \begin{bmatrix} e_1(t) \\ e_2(t) \end{bmatrix}
\]

Or

\[
\begin{bmatrix} z_1(t) \\ z_2(t) \end{bmatrix} = \phi_{01} \begin{bmatrix} z_1(t-1) \\ z_2(t-1) \end{bmatrix} + \phi_{11} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} z_1(t-1) \\ z_2(t-1) \end{bmatrix} + \begin{bmatrix} e_1(t) \\ e_2(t) \end{bmatrix}
\]

Equation (1) can be stated in a form:

\[
\begin{align*}
z(t) &= \left[ \phi_{01} I + \phi_{11} W \right] z(t-1) + e(t) \\
z(t) &= z(t-1) + e(t) \\
z(t) &= \left[ \phi_{01} I + \phi_{11} W \right] z(t-1) + e(t)
\end{align*}
\]

with

\[
\Phi = \left[ \phi_{01} I + \phi_{11} W \right]
\]

Next, validation model is conducted through error. If the model determined as adequate, then it can be used to forecast future observation by using linear equation model [17]:

\[
y = X \beta + e(t), e(t) \overset{iid}{\sim} N(0, \sigma^2)
\]  

STAR(1;1) model can be written

\[
z(t) = [z(t-1), Wz(t-1)] \begin{bmatrix} \phi_{01} \\ \phi_{11} \end{bmatrix} + e(t)
\]  

and

\[
y = X \beta + e(t)
\]
Thus, STAR (1;1) model parameter estimation is obtained by using the smallest square method
\[
\hat{\beta} = \left( \hat{\phi}_{01} \ \hat{\phi}_{11} \right) = \left( X' X \right)^{-1} X' \tilde{y}.
\] (7)

with
\[
X = [z(t-1) \ \ Wz(t-1)]
\]

Matrix X above is a 2x2 size matrix contains:
\[
X = \begin{bmatrix}
z_1(t-1) & 0 \\
z_2(t-1) & 1
\end{bmatrix}
\]
\[
\begin{bmatrix}
z_1(t-1) \\
z_2(t-1)
\end{bmatrix}
\]

Theorem: STAR(1;1) is said to be stationary, if it meets the following condition;
\[
|\phi_{01}| + |\phi_{11}| < 1
\] (9)

[4] in [2] has proven the stationary requirement as follows:
\[
|x_u I - \phi_{10} I - \phi_{11} W| = 0
\] (10)
or
\[
|\phi_{11} W - (\phi_{10} - x_u) I| = 0
\]

with \(\lambda(x_u) = \phi_{10} - x_u\)
and \(\lambda(x_u)\) is Eigenvalue \(A = -\phi_{11}\)

Each Eigenvalue A located in one of closed unit circles, stated by
\[
|x - a_{ij}| \leq \sum_{j=1}^{N} |a_{ij}|
\]

For STAR (1;1) model case, the application of calculus theorem conducted by taking \(A = -\phi W\) and
\[
\sum_{j=1}^{N} |a_{ij}| = \sum_{j=1}^{N} w_{ij} |\phi_{11} x| = |\phi_{11}|
\]
\(w_{ij} = 0\)
\(\sum_{j=1}^{N} w_{ij} = 1\)
thus
\[
a_{ij} = \phi_{11}\]
can be written as:

\[ |\lambda - 0| = |\phi_{11}| \]

And because all circles are the same, thus all Eigenvalues need to meet:

\[ |\lambda| \leq |\phi_{11}| \]

For STAR(1;1) model \( \lambda(x_u) = \phi_{10} - x_u \)

therefore

\[ |\phi_{10} - x_u| \leq |\phi_{11}| \]

(11)

equation (2) is the same as follows

\[ |\phi_{10} - x_u| \leq |\phi_{11}| \]

and

\[ x_u - \phi_{10} \leq |\phi_{11}| \]

thus

\[ \phi_{10} - |\phi_{11}| \leq x_u \leq \phi_{10} + |\phi_{11}| \]

because

\[ |x_u| < 1 \text{ or } -1 < u < 1, \]

thus

\[ -1 < \phi_{10} - |\phi_{11}| \leq x_u \leq \phi_{10} + |\phi_{11}| < 1 \]

or

\[ \phi_{10} - |\phi_{11}| > -1 \]

and

\[ \phi_{10} - |\phi_{11}| < 1 \]

Both conditions combined, so that the STAR (1;1) model will be stationary with

\( (\phi_{10}, \phi_{11}) \) parameter meets

\[ |\phi_{10}| + |\phi_{11}| < 1 \]

(12)
STAR(1;1) model on two petroleum wells production data in Balongan Indramayu West Java

Table Petroleum Data

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Summary

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<td>3rd Qu.:</td>
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<td>Max.:</td>
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Cor
The stationary requirement for STAR(1;1) model is fulfilled, because STAR (1;1) model parameter estimation which is \( \phi_{01} \) and \( \phi_{11} \) values as follows:

\[
\phi_{01} = 0.91697 \\
\phi_{11} = 0.07295
\]
Implying stationary requirement is fulfilled as follows

$$|\phi_{01}| + |\phi_{11}| < 1$$

This value indicates that STAR (1;1) model on petroleum production data is stationary. Those values obtained by using Microsoft Excel.

STAR model (Time Series) (Excel 2003)
matrix phi

\[
\begin{align*}
\text{phi}_{01} &= 0.91697 \\
\text{phi}_{11} &= 0.072946
\end{align*}
\]

Stationary: 0.989916

Image 3: Error graph of 2 petroleum wells

4. Conclusion
STAR(1;1) model on two petroleum wells data is stationary and meets the requirement.

$$|\phi_{01}| + |\phi_{11}| < 1$$

References

and Culex quinquefasciatus habitats based on spatiotemporal field-sampled count data. *Acta tropica* 117:61-8


