A productive exposition for the thermal post buckling problem of orthotropic circular plates by using relevant admissible function for lateral displacement

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Abstract: A suitable mathematical formulation is used to study the thermal post buckling problem of orthotropic circular plates is presented herein. The total energy equation is reduced by assuming algebraic function for lateral displacement w. The radial tension per unit length is calculated by substituting the value of orthotropic parameter under simply supported and clamped boundary conditions. The thermal post buckling load carrying capacity of the plate is evaluated using integrated average for various values of orthotropic parameter β, and the numerical results obtained from the present investigation are compared with the results obtained from the known literatures. The results obtained for both cases are concurs well with reference values and the numerical results using integrated average shows more accurate values for the post buckling load.

Keywords: Thermal post buckling; von Karman nonlinearity; orthotropic; circular plates; radial tension, simply supported, clamped

INTRODUCTION

The plate structures of isotropic or orthotropic nature is an important research topic because of its relation to ocean, aerospace, mechanical and civil engineering applications. Circular plates are extensively used in sensors and other elemental structures in engineering, the investigations on the behavior of mechanics such as deformation, vibration and buckling. Problems of vibrations and post buckling of circular plates have been studied by many authors using different methods such as finite element method, Berger's approximations, shooting method, shear deformation theory etc.
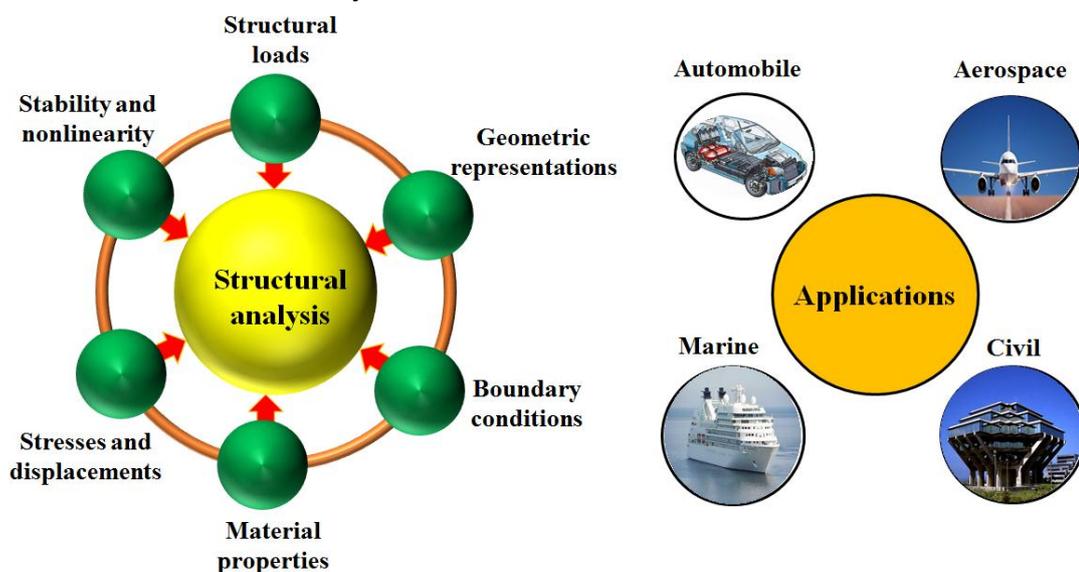
Figure 1. Structural analysis diagram and its applications

Nonlinear analysis of structural members with effects of geometric and material nonlinearity is of research importance, as the results obtained by the analysis leads to meaning-

ful yet accurate results in lesser times. Most of the earlier studies have been using the classical methods/energy methods/variational methods and the versatile finite element method. However, the solution procedure in all the studies mentioned above involves in solving higher mathematical terms and solving higher order differential equations or involve in cumbersome calculations. Structural members when subjected to compressive loads are prone to an instability phenomenon called buckling (Wang & Wang, 2004). According to the linear theory of buckling, the structures are treated as functionally failed, when the compressive load reaches the buckling load. With the present day, highly optimum and cost effective designs in structural engineering, it is not necessary to treat that the structure fails at the buckling load, contrary to the popular belief. The inherent geometric nonlinearity involved gives an additional load carrying capacity to the structures after phenomenon of buckling (Dym & Hsu, 1975; Pearson, 1956). The structures are capable of taking additional compressive loads with high deformations. But, if these deformations are tolerable and do not effect the functional requirements, the additional load carrying capacity of these structure, called as the post buckling load, can be advantageously used in the design process. Further, the thermal post buckling load, due to a temperature rise from the stress free temperature, in the service condition of these structures, is an order of higher in magnitude than the mechanical loads as seen in the literature (Dym & Hsu, 1975; Pearson, 1956). This property can be used advantageously for aerospace and other structures subjected to thermal loads due to the temperature raise.

The post buckling behavior of orthotropic circular plates evaluated using finite element analysis has been described in (Kanaka Raju & Venkatewara Rao, 1983). Rao and Varma (G Venkateswara Rao & Varma, 2007) used a simple formulation to predict the thermal post buckling capacity of uniform, isotropic circular plates by finding the edge tensile load with simply supported and clamped boundary conditions. This formulation reinvestigated by using an intuitive approach for evaluating the thermal post buckling load carrying capacity of circular plates in (Ramaraju & Gundabathula, 2009). Li *et al.* (Li, Batra, & Ma, 2007) studied the vibrations of prebuckled and thermally post buckled polar orthotropic plates with simply supported and clamped boundary conditions by using shooting method. The effects of temperature rise and the boundary conditions on plate's frequencies have been analyzed. Using Rayleigh – Ritz method, Mazhari and Shahidi (Mazhari & Shahidi, 2011) investigated the post buckling behavior of circular homogenous plates with concentric holes under uniform radial loading. Varma and Rao (Varma & Rao, 2011) presented a novel formulation to study the thermal post buckling behavior of uniform thin circular plates with an edge rotational restraint on the basis of the radial edge tensile load. By applying finite element formulation, Raju and Rao (Raju & Rao, 1985) discussed the post buckling behavior of linearly tapered orthotropic circular plates with different tapers. The post buckling behavior of elastic circular plates using simple finite element formulation has been presented by Rao and Raju (G Venkateswara Rao & Raju, 1979). For reducing the errors, the authors (G Venkateswara Rao & Raju, 1983) reinvestigated the problem of post buckling behavior by evaluating the nonlinear stiffness matrix using finite element formulation. The thermal post buckling behavior of circular plates with an edge rotational restraint has been investigated by Rao et.al (G. Venkateswara Rao, Raju, & Naidu, 1992) using finite element analysis.

The influence of orthotropic parameters on the large amplitude response of circular plates with both simply supported and clamped boundary conditions has been studied in (Nath & Alwar, 1980). The buckling problem of tapered orthotropic circular plates has been investigated using computationally economic finite element method in (Bhushan, Singh, & Rao, 1993). The effects of numerous parameters such as boundary conditions, modular ratios and tapers for linear buckling load has been studied. In all these studies, the prediction of the post buckling load contains solving either the nonlinear differential equations or obtaining approximate solutions from the nonlinear energy formulations and its complexity is high. Because of the coupling between the radial and strain components, the calculation of radial edge tensile load is very difficult from the nonlinear governing equations.

Thin circular plates are commonly used structural members in large aerospace structures. During their service condition, these plates are subjected to thermal loads, arising from the aero-

dynamic and/or solar heating. Prediction of the thermal post buckling load of such plates serves the purpose of arriving at the cost effective designs as mentioned earlier. However, the circular plates pose a difficulty, because of the explicit coupling between the radial and circumferential strains. As such, the tension developed in the plate configuration due to large deflections is not easier for derivation, unless some assumptions or approximations are made. A new mathematical approximation technique to study the thermal post buckling behavior of orthotropic circular plates is described in this manuscript. The radial edge tensile load is evaluated by assuming suitable admissible function for the lateral displacement 'w', using substitution method based on von Karman nonlinearities and hence calculated the thermal post buckling load carrying capacity for various values of orthotropic parameter β. The present formulation requires only the knowledge of radial edge tensile load developed due to the large lateral displacements and linear buckling load parameter. Furthermore, a comparative study is carried out for the post buckling problem with and without using integrated average which gives a clear information about the results of this formulation.

METHODS

Consider an orthotropic circular plate of radius a and constant thickness h with an external radial uniform compressive load N_r per unit length at the boundary.
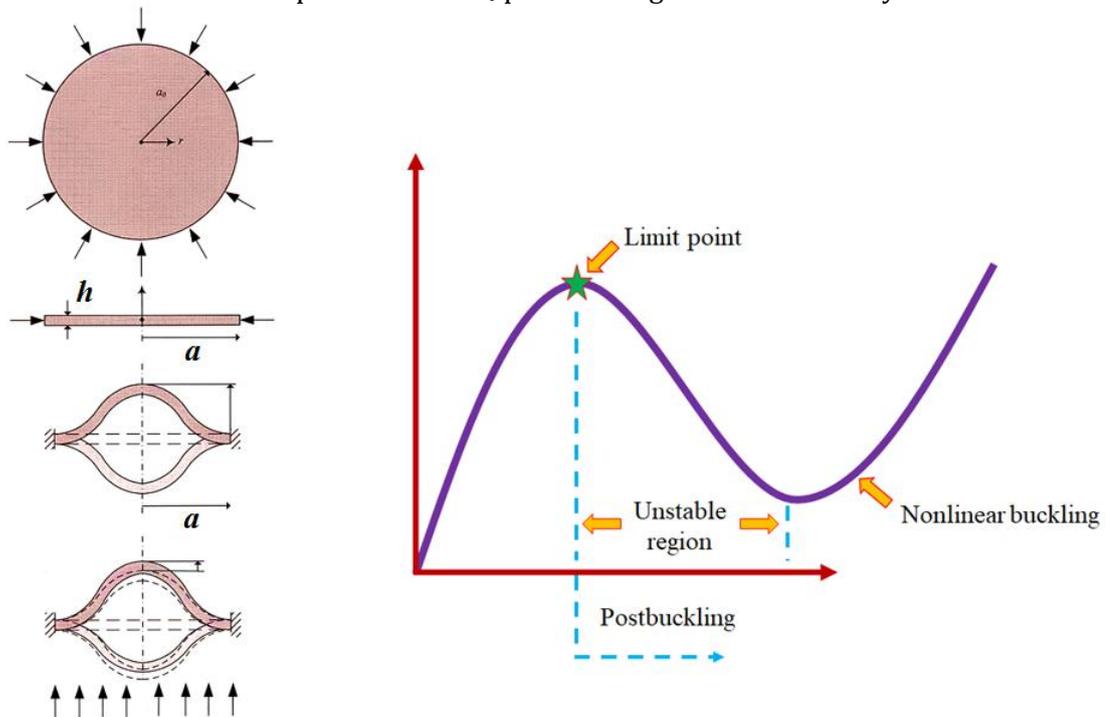
Figure 2. The buckling and post buckling of circular plate (Williams, Griffin, Homeijer, Sankar, & Sheplak, 2007) and its graphical representation.

The strain displacement relations of orthotropic circular plate for axisymmetric conditions are given by

$$\varepsilon_r = \frac{du}{dr} + \frac{1}{2}\left(\frac{dw}{dr}\right)^2 \tag{1}$$

$$\varepsilon_\theta = \frac{u}{r} \tag{2}$$

$$\chi_r = -\frac{d^2w}{dr^2} \tag{3}$$

$$\chi_\theta = -\frac{1}{r}\left(\frac{dw}{dr}\right) \tag{4}$$

By using the above mentioned relations, the strain energy U, of the orthotropic circular plate can be defined as

$$U = \frac{1}{2}\int_0^{2\pi}\int_{r_1}^{r_2}\left[C_1\varepsilon_r{}^2 + C_2\varepsilon_\theta{}^2 + C_{12}\varepsilon_r\varepsilon_\theta + D_1\chi_r{}^2 + D_2\chi_\theta{}^2 + D_{12}\chi_r\chi_\theta\right] r\, dr\, d\theta \tag{5}$$

where $C_1 = \dfrac{E_r h}{1-\nu^2}$, $C_2 = \dfrac{E_\theta h}{1-\nu^2}$, $C_{12} = \nu_r C_2 = \nu_\theta C_1$, $D_1 = \dfrac{E_r h^3}{12\left(1-\nu^2\right)}$,

$D_2 = \dfrac{E_\theta h^3}{12\left(1-\nu^2\right)}$ and $D_{12} = \nu_r D_2 = \nu_\theta D_1$.

By substituting the values of C_1, C_2, C_{12}, D_1, D_2, D_{12}, then equation (5) will become

$$U = \frac{1}{2}\int_0^1 [\, \frac{E_r h}{1-\nu_r\nu_\theta}\left[\frac{du}{dr}+\frac{1}{2}\left(\frac{dw}{dr}\right)^2\right]^2 + \frac{E_\theta h}{1-\nu_r\nu_\theta}\left(\frac{u}{r}\right)^2 + \nu_r\cdot\frac{E_\theta h}{1-\nu_r\nu_\theta}\left[\frac{du}{dr}+\frac{1}{2}\left(\frac{dw}{dr}\right)^2\right]\left(\frac{u}{r}\right)$$
$$+ \frac{E_r h^3}{12(1-\nu_r\nu_\theta)}\left(-\frac{d^2w}{dr^2}\right)^2 + \frac{E_\theta h^3}{12(1-\nu_r\nu_\theta)}\left[\frac{1}{r}\cdot\frac{dw}{dr}\right]^2 + \nu_r\cdot\frac{E_\theta h^3}{12(1-\nu_r\nu_\theta)}\cdot\frac{-d^2w}{dr^2}\cdot\frac{1}{r}\cdot\frac{-dw}{dr}\,]\, r\, dr \tag{6}$$

When u = 0, the equation (6) becomes

$$U = \frac{1}{2}\frac{E_r h}{1-\nu_r\nu_\theta}\int_0^1\left[\frac{1}{4}\left(\frac{dw}{dr}\right)^4 + \frac{h^2}{12}\left(\frac{d^2w}{dr^2}\right)^2 + \frac{\beta}{r^2}\cdot\frac{h^2}{12}\left(\frac{dw}{dr}\right)^2 + \nu_r\cdot\frac{\beta}{r}\cdot\left(\frac{d^2w}{dr^2}\right)\left(\frac{dw}{dr}\right)\right] r\, dr \tag{7}$$

After substitution of the values and elimination of h, equation (7) can be reduced as shown in equation (8).

$$U = \frac{1}{2}\int_0^1\left(\frac{d^2w}{dr^2}\right)^2 + \beta\frac{1}{r^2}\left(\frac{dw}{dr}\right)^2 + 2\nu\frac{\beta}{r^2}\left(\frac{d^2w}{dr^2}\right)\left(\frac{dw}{dr}\right) dr \tag{8}$$

where $\beta = \dfrac{E_\theta}{E_r}$ is the orthotropic parameter with $E_\theta \neq E_r$.

The work done due to the external compressive load N_r per unit length at the boundary is given by

$$W = \frac{1}{2}\int_0^{2\pi}\int_{r_1}^{r_2}\overline{N}_r\left(\frac{dw}{dr}\right)^2 r\, dr\, d\theta \tag{9}$$

where $\overline{N}_r$ is the radial load distribution per unit length.

By using substitution for $\overline{N}_r$, equation (9) can be written in terms of β and λ as follows.

$$W = \frac{\lambda\beta}{2}\int_0^1\frac{d^2w}{dr^2}dr \tag{10}$$

The total potential energy of the orthotropic circular plate can be indicated as
$$\Pi = U - W \tag{11}$$

The equation of total energy is reduced using Rayleigh – Ritz method included in (G Venkateswara Rao & Raju, 2002; Venkateswara Rao & Kanaka Ruju, 1984) in view of the assumed displacement functions. This paper describes the minimization of total potential energy by assuming algebraic function for lateral displacement w.

The way of finding radial edge tensile load and hence the thermal post buckling load carrying capacity is discussed in the preceding section. The suitable admissible function for the lateral displacement w, which satisfies the geometric boundary conditions is used to find the radial edge tensile load. The value of Poisson's ratio ν is taken as 0.3. The orthotropic parameter

β is ranging from 1.2 to 2.0 in steps of 0.2 with simply supported and clamped boundary conditions are considered. The assumed algebraic function used in this study can be defined as

$$F = b_0 \left(1 - \left(\frac{r}{a} \right)^2 \right)^n \tag{12}$$

The function F satisfies the geometric boundary conditions and the values of $n = 1$ and $n = 2$ depict the simply supported and clamped boundary conditions respectively.

Simply supported:
At r = 0 , $w' = 0$;   At r = a, $w = 0$

Clamped:
At r = 0, $w' = 0$;   At r = a, $w = 0$, $w' = 0$

An equivalent uniform compressive edge load N_r is established in when the plate is heated to a temperature ΔT from the initial stress free state. As the temperature is increased to critical temperature (ΔT_r), buckling of the plate occurs owing to the development of critical compressive uniform radial edge load $N_{r_{cr}}$. A uniform tensile edge load N_{r_T} is developed as a result of high lateral displacements arose due to a further increase in temperature (ΔT); allowing the plate to withstand more equivalent compressive radial edge load (N_r) beyond N_{cr}. Owing to the stated reasons, the total equivalent compressive uniform radial edge load carrying capacity of the circular plate ($N_{r_{NL}}$) or post buckling load can be expresses in non-dimensional mathematical form as

$$\overline{N}_{r_{NL}} = \overline{N}_{r_{Cr}} + \overline{N}_{r_T} \tag{13}$$

where each term in equation (13) is non – dimensionalised as

$$\overline{N}_r = \frac{N_r a^2}{D} \tag{14}$$

and D is the plate flexural rigidity.

As compared to the square plates (G Venkateswara Rao & Raju, 2004) it is very difficult to evaluate the tension parameter for the circular plates because of the coupling between the radial and circumferential strains by the radial displacement u. From equations (1) and (2), the radial tension per unit length can be taken as

$$N_r = \frac{\beta t}{1 - v^2} \left[\frac{du}{dr} + \frac{1}{2} \left(\frac{dw}{dr} \right)^2 + v \frac{u}{r} \right] \tag{15}$$

By using the approximation proposed by Berger (Berger, 1954), which states that the second invariant of the strains are neglected or $\varepsilon_r \ll \varepsilon_\theta$, N_r can be written as

$$N_r = \frac{\beta t}{1 - v^2} \left[\frac{du}{dr} + \frac{1}{2} \left(\frac{dw}{dr} \right)^2 \right] \tag{16}$$

By referring the work of Wah (Wah, 1963) given by Leissa (Leissa, 1969), the edge tensile load N_r is considered as constant and hence

$$\frac{dN_r}{dr} = 0 \tag{17}$$

Or $\dfrac{d^2 u}{dr^2} = - \left(\dfrac{dw}{dr} \right) \left(\dfrac{d^2 w}{dr^2} \right)$ (18)

By integrating equation (18) twice, u can be evaluated for the selected admissible function. The constants of integration are acquired using the boundary conditions on radial displacement.

Once the functional form of u is known, a better approximation for the radial tension N_r, which is still treated as a constant, along the radius is obtained as

$$N_r = \frac{\beta t}{\left(1 - v^2 \right)} \left[\frac{du}{dr} + \frac{1}{2} \left(\frac{dw}{dr} \right)^2 + v \left(\frac{u}{r} \right)_{av} \right] \tag{19}$$

In equation (19), to make N_r constant along the radius, the integrated average of $\varepsilon_\theta = \left(\dfrac{u}{r}\right)_{av}$ is given by

$$\left(\frac{u}{r}\right)_{av} = \left(\varepsilon_\theta\right)_{av} = \frac{1}{a}\int_0^a \frac{u}{r}\, dr \tag{20}$$

By using values of edge tensile load parameters, the thermal post buckling load carrying capacity of the orthotropic circular plates can be evaluated for different values of orthotropic parameter β.

The post buckling load of plates is given in terms of $N_{r_{NL}}$ and $N_{r_{cr}}$ as

$$N_{r_T} = \frac{12}{\beta\left(1-v^2\right)}\int_0^1 \left(\frac{dw}{dr}\right)^2 dr \tag{21}$$

$$\gamma = \frac{N_{r_{NL}}}{N_{r_{cr}}} = 1 + c\left(\frac{b_0}{h}\right)^2 \tag{22}$$

RESULTS AND DISCUSSION

From equation (19), the radial edge tensile load N_r for orthotropic circular plate can be evaluated by assuming suitable admissible function for the lateral displacement w. The boundary of the plate is assumed to be simply supported and clamped. The thermal post buckling load carrying capacity is calculated using the algebraic assumed function which satisfies both the boundary conditions. The γ values for various orthotropic parameters β ranging from 1.2 to 2.0 are evaluated considering the integrated average. The numerical results are obtained in both cases are compared with results obtained from the known literatures.

Table 1. Representing values of post buckling load carrying capacity 'γ' of simply supported circular plates for the assuming function $F = b_0\left[1-\left(\dfrac{r}{a}\right)^2\right]^n$

$\dfrac{b_0}{h}$	Present results									
	β = 1.2	Error (%)	β = 1.4	Error (%)	β = 1.6	Error (%)	β = 1.8	Error (%)	β = 2.0	Error (%)
0.0	2.2624	0.30	2.2554	0.39	2.2469	0.39	2.2356	0.25	2.2284	0.27
	(2.2786)*	1.01	(2.2621)*	0.69	(2.2518)*	0.61	(2.2426)*	0.57	(2.2354)*	0.59
	(2.2557)**		(2.2466)**		(2.2381)**		(2.2300)**		(2.2224)**	
0.2	2.2751	0.40	2.2667	0.45	2.2565	0.39	2.2457	0.29	2.2385	0.32
	(2.2879)*	0.96	(2.2718)*	0.67	(2.2642)*	0.73	(2.2507)*	0.51	(2.2431)*	0.52
	(2.2661)**		(2.2566)**		(2.2478)**		(2.2393)**		(2.2314)**	
0.4	2.3065	0.40	2.2973	0.47	2.2827	0.26	2.2709	0.16	2.2596	0.05
	(2.3146)*	0.75	(2.3014)*	0.65	(2.2875)*	0.48	(2.2761)*	0.39	(2.2614)*	0.13
	(2.2972)**		(2.2866)**		(2.2767)**		(2.2673)**		(2.2584)**	
0.6	2.3506	0.08	2.3396	0.14	2.3267	0.09	2.3166	0.03	2.3092	0.26
	(2.3553)*	0.27	(2.3476)*	0.48	(2.3295)*	0.21	(2.3205)*	0.3	(2.3198)*	0.72
	(2.3487)**		(2.3363)**		(2.3246)**		(2.3136)**		(2.3032)**	
0.8	2.4328	0.52	2.4102	0.20	2.3995	0.34	2.3792	0.05	2.3666	0.05
	(2.4536)*	1.37	(2.4146)*	0.38	(2.4142)*	0.95	(2.3817)*	0.16	(2.3692)*	0.16
	(2.4203)**		(2.4054)**		(2.3913)**		(2.3780)**		(2.3654)**	
1.0	2.5202	0.35	2.5014	0.33	2.4862	0.41	2.4665	0.26	2.4548	0.41
	(2.5283)*	0.67	(2.5085)*	0.61	(2.4976)*	0.87	(2.4831)*	0.94	(2.4629)*	0.74
	(2.5114)**		(2.4932)**		(2.4761)**		(2.4600)**		(2.4448)**	

* Indicates the value of thermal post buckling load carrying capacity calculated without using integrated average (Nair, Kasim, & Salleh, 2017)
** Indicates the reference values taken from (Kanaka Raju & Venkatewara Rao, 1983).

Table 1 present the thermal post buckling load carrying capacity (γ) of simply supported orthotropic circular plates for different values of β ranging from 1.2 to 2.0. The numerical results evaluated from the present investigation is compared with the results from (Kanaka Raju & Venkatewara Rao, 1983; Nair et al., 2017). Besides, a comparative study has been carried out between the present values and known literature values. From this investigation it can be found that the values of γ calculated using integrated average in the present formulation is more accurate. In this study the maximum error percentage obtained for simply supported boundary conditions is 1.37% which is less than the values without using integrated average (Nair et al., 2017).

Table 2. Representing values of post buckling load carrying capacity 'γ' of clamped circular plates

for the assuming function $F = b_0 \left(1 - \left(\dfrac{r}{a} \right)^2 \right)^n$

| $\dfrac{b_0}{h}$	Present results									
	$\beta = 1.2$	Error (%)	$\beta = 1.4$	Error (%)	$\beta = 1.6$	Error (%)	$\beta = 1.8$	Error (%)	$\beta = 2.0$	Error (%)
0.0	2.2125 (2.2237)* (2.1951)**	0.79 1.30	2.2095 (2.2145)* (2.1924)**	0.78 1.01	2.2064 (2.2095)* (2.1894)**	0.78 0.92	2.1912 (2.1988)* (2.1860)**	0.24 0.59	2.1852 (2.1876)* (2.1824)**	0.13 0.24
0.2	2.2236 (2.2515)* (2.2031)**	0.93 2.19	2.2192 (2.2416)* (2.2003)**	0.86 1.88	2.2142 (2.2300)* (2.1972)**	0.77 1.49	2.2102 (2.2195)* (2.1936)**	0.76 1.18	2.1974 (2.2084)* (2.1899)**	0.34 0.85
0.4	2.2461 (2.2865)* (2.2271)**	0.85 2.67	2.2398 (2.2728)* (2.2239)**	0.71 2.19	2.2365 (2.2673)* (2.2204)**	0.73 2.11	2.2329 (2.2529)* (2.2164)**	0.74 1.65	2.2272 (2.2373)* (2.2122)**	0.68 1.14
0.6	2.2899 (2.3356)* (2.2666)**	1.03 3.04	2.2791 (2.3210)* (2.2629)**	0.72 2.57	2.2764 (2.3122)* (2.2587)**	0.78 2.37	2.2748 (2.2964)* (2.2540)**	0.92 1.88	2.2668 (2.2781)* (2.2491)**	0.79 1.29
0.8	2.3526 (2.3928)* (2.3212)**	1.35 3.08	2.3417 (2.3799)* (2.3167)**	1.08 2.73	2.3369 (2.3681)* (2.3117)**	1.09 2.44	2.3306 (2.3552)* (2.3061)**	1.06 2.13	2.3261 (2.3333)* (2.3002)**	1.13 1.44
1.0	2.4235 (2.4557)* (2.3902)**	1.39 2.74	2.4182 (2.4386)* (2.3848)**	1.40 2.26	2.4061 (2.4278)* (2.3788)**	1.15 2.06	2.3915 (2.4153)* (2.3720)**	0.82 1.83	2.3759 (2.3951)* (2.3648)**	0.47 1.27

* Indicates the value of thermal post buckling load carrying capacity calculated without using integrated average (Nair et al., 2017).
** Indicates the reference values taken from(Kanaka Raju & Venkatewara Rao, 1983).

The corresponding results for clamped circular plates are presented in Table 2. The values of γ are evaluated for different orthotropic parameters with various $\dfrac{b_0}{h}$ values. As in the case of simply supported boundary conditions, the γ values calculated using integrated average give the closer values than (Nair et al., 2017) when compared to reference values. The maximum error percentage for clamped case is 2.67% which are matching well with the reference values taken from literature within the engineering accuracy. It is assumed in the Berger's approximation that the strain energy due to the second variant of the middle surface strains can be neglected could be the reason for the much higher percentage errors for both the simply supported and clamped circular plate.

The values of γ evaluated using integrated average for various $\dfrac{b_0}{h}$ ranging from 0 to 1 by the assumed algebraic function F under simply supported and clamped boundary conditions are plotted against various values of orthotropic parameters β ranging from 1.2 to 2.0 are illustrated in Figure 2. From the graph, it can be shown that the results obtained from the present

formulations and those given by the literature follow the same trend with a percentage error of 1.37% to 2.67%.
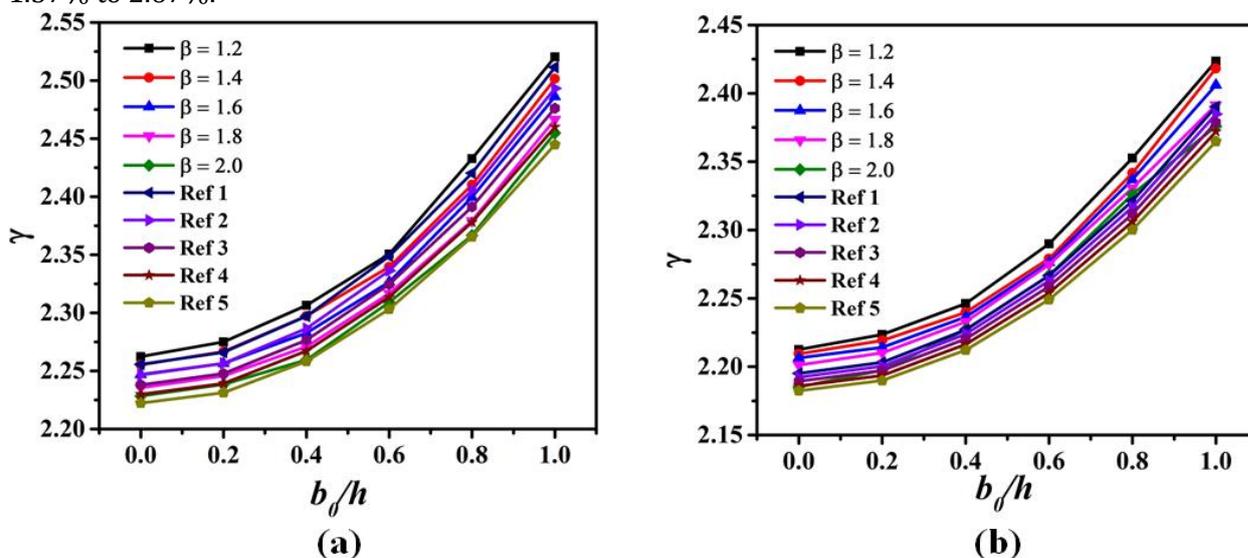
Figure 2. Thermal post buckling load carrying capacity of orthotropic circular plates for various $\frac{b_0}{h}$ against various values of orthotropic parameters β under simply supported (a) and clamped (b) boundary conditions. Ref (1-5) denotes the reference values from (Kanaka Raju & Venkatewara Rao, 1983).

The post buckling problem of orthotropic plates are solved earlier using different mathematical formulations such as finite element approach, iterative methods and so on. The present mathematical method helps to reduce the complexity of the solution procedure and improve the accuracy of the results. The substitution method is used to reduce the difficulty of solving the governing differential equations which helps to find the thermal post buckling load carrying capacity of circular plates for various β values.

CONCLUSIONS

A suitable mathematical formulation in obtaining the thermal post buckling load carrying capacity of orthotropic circular plates using integrated average is presented in this paper by reducing the complexity of evaluating the edge tensile load. Based on von Karman nonlinearity, the differential equations of total potential energy is integrated to evaluate the linear buckling load and an algebraic admissible function for the lateral displacement is used to calculate the radial edge tensile load. The linear buckling load parameters and radial edge tensile load are used to determine the post buckling load of orthotropic circular plates for various values of orthotropic parameter β. The numerical results obtained in this study match well with those obtained from the known literatures within the engineering accuracy.

ACKNOWLEDGEMENT

NOMENCLATURE

a = radius of the circular plate
E = Young's modulus
N_r = uniform radial edge compressive load per unit length
$N_{r_{cr}}$ = linear buckling load

$N_{r_{NL}}$ = total uniform radial edge compressive load per unit length

N_{r_T} = uniform radial edge tensile load per unit length developed due to large lateral displacements

H = thickness of the plate

ΔT = temperature rise from the stress free temperature

$\varepsilon_r , \varepsilon_\theta$ = in – plane strains

ν = Poisson's ratio

U   = strain energy

r, θ = radial and circumferential coordinates

W   = work done

χ_r , χ_θ = curvatures

r_1, r_2 = internal and external radii

$\alpha_1 - \alpha_6$ = generalized displacements

β = orthotropic parameter

E_r , E_θ = Young's moduli in radial and circumferential directions

λ = linear buckling load parameter

γ = postbuckling load

b_0 = central (maximum) lateral displacement of the circular plate

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